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Prediction of nucleate pool boiling heat transfer coefficients for binary mixtures

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INTRODUCTION

THE BOILING of binary mixtures is of practical significance for chemical engineering and heat pump applications. So far, no predictive equations of broad generality for the determination of nucleate pool boiling heat transfer coefficients for binary mixtures have appeared in the literature, although numerous experimental investigations were reported. This fact was also mentioned in a recent paper [1]. The object of this study is the derivation of a correlation to determine these heat transfer coefficients. For a detailed literature survey on the subject, the reader is directed to refs. [2, 3]. Only the literature data found to be pertinent to this study will be mentioned.

Sufficient empirical evidence was given in the literature to the effect that, for a given heat flux and pressure, the nucleate pool boiling heat transfer coefficient for a binary mixture can be considerably lower than the molar average of the nucleate pool boiling heat transfer coefficients for the pure components of the mixture. Van Wijk *et al.* [4] gave the following explanation for this heat transfer deterioration: the bubbles leaving the heated surface are enriched in the volatile component (i.e. lower boiling point component). This results in a reduction of this particular component in the boiling boundary layer in the vicinity of the heated surface. The liquid mole fraction of the volatile component in this layer is therefore lower than that in the bulk liquid. Consequently, the boiling temperature in the layer becomes higher than that in the bulk liquid. This can be deduced from a vapour–liquid phase equilibrium diagram of a typical binary mixture. For nucleate pool boiling, the heat transfer coefficient is a function of the wall superheat, i.e. the difference between the wall temperature and the liquid boiling temperature. For the determination of this heat transfer coefficient for a binary mixture, the measured wall superheat (based on bulk liquid boiling temperature) is used, whilst the wall superheat in the boiling boundary layer is driving the flow of heat. The latter is smaller than the former.

The following explanations were also given to clarify the quoted deterioration in heat transfer:

- the change in bubble growth rate caused by the varying resistance to mass transfer of the volatile component in diffusing into a growing bubble [5];
- the increase of wall superheat required to activate bubble nucleation centres for mixtures, resulting in a less-dense bubble population at a given wall superheat as compared with that for pure liquids [6];
- the retardation of two of the three principal heat transport mechanisms (i.e. vapour–liquid exchange and evaporative mechanisms) active in nucleate pool boiling for mixtures [7].

The most popular correlation for the prediction of ΔT_m, the wall superheat at a given heat flux during nucleate pool boiling of a binary mixture, is from Stephan and Körner [8]. This wall superheat is given by the equation:

$$\Delta T_m/\Delta T_i = [1+K\,|y-x|(0.88+0.12\times 10^{-5}P)] \quad (1)$$

where K is an empirical constant different for every binary mixture. ΔT_i in equation (1), the ideal wall superheat, and the

NOMENCLATURE

a	thermal diffusivity of liquid [$m^2\,s^{-1}$]
$b_1,\ldots,b_5$	functions defined in the text
c_p	liquid specific heat [$J\,kg^{-1}\,K^{-1}$]
D	liquid mass diffusivity [$m^2\,s^{-1}$]
h	heat transfer coefficient [$W\,m^{-2}\,K^{-1}$]
K	empirical constant
n	number of data
P	pressure [Pa]
P_c	critical pressure of volatile component [Pa]
q	heat flux [$W\,m^{-2}$]
r	latent heat of evaporation [$J\,kg^{-1}$]
Sn	Scriven number
T	bulk liquid boiling temperature [K]
ΔT	wall superheat, i.e. the difference between wall and bulk liquid boiling temperature [K]
x	mole fraction of liquid for volatile component
y	mole fraction of vapour for volatile component.

Subscripts

1	pure non-volatile component
2	pure volatile component
i	ideal binary mixture
m	real binary mixture
w	evaluation for mass fraction basis.

heat transfer coefficient are respectively,

$$\Delta T_i = \Delta T_1(1-x)+\Delta T_2 x \qquad (2)$$

and

$$h = q/\Delta T_m \qquad (3)$$

where ΔT_m in equation (3) is determined from equation (1). $|y-x|$ in equation (1) is the mass transfer driving force. Stephan and Körner [8] showed that energy required to form a vapour bubble in a binary mixture is a function of this force. These investigators determined the empirical constant K in equation (1) for 17 binary mixtures. This equation is only valid for a limited pressure-range between 0.1 and 1 MPa [8]. Stephan and Körner reported that the error in predicting the heat transfer coefficient with equation (3) from the data taken with these mixtures was within 22% with a confidence level of 99%: the r.m.s. error obtained was 8.6% [8, 9]. It appears from Körner's thesis [9], however, that this equation was not verified with the data taken with the smaller values of the liquid mole fraction of the volatile component, say about $x < 0.021$. If such data are also considered, then the error in predicting the heat transfer coefficient with the equation is, in fact, considerably higher than that previously mentioned; the equation predicts the heat transfer coefficient from the 321 data reported in Körner's thesis [9] for $P < 1$ MPa (see lines 1–8 in Table 1) accurate to 22% for 93% of the time. For the rest of the time however, the equation yields the heat transfer coefficients with deviations ranging from 22% to 107.8%. The r.m.s. error for these 321 data is 15.1%. If the data of refs. [10, 11] and those of Shakir, as quoted in ref. [1], are also taken into account (see lines 9 and 10 in Table 1), then the r.m.s. error obtained for the 357 data analysed is 16.8%, whilst the equation is accurate to 22% for 89% of the time. An error is based on the measured value and the r.m.s. error is the standard deviation of the errors.

One of the apparent disadvantages in using equation (1) for practical applications, is that experimental data are needed to determine the empirical constant K therein at a given pressure range for a particular binary mixture. In the absence of experimental data and for $|y-x| < 0.635$ and $P = 0.1$–1 MPa, Stephan and Körner [8] recommended an average value of 1.53 for K. They reported that the r.m.s. error in predicting heat transfer coefficients was about 15% if $K = 1.53$. Using this K value and the 357 data already mentioned, the error in predicting the heat transfer coefficient with equation (3) was calculated. This equation predicts the heat transfer coefficient from these data accurately to 32% (i.e. about twice the r.m.s. error) for 97% of the time. For the rest of the time, the error varies from 32% to 106.7%. The r.m.s. error for the 357 data is 15.9%.

Comparing two expressions derived by Scriven [12] for vapour bubble growth in a pure liquid and in a binary mixture and using the modified Jakob number as suggested by Van Stralen [13], Calus and Leonidopoulos [10] obtained the wall superheat for the nucleate pool boiling of a binary mixture as:

$$\Delta T_m = (\Delta T_i/Sn)_w \qquad (4)$$

where, Sn, the Scriven number, is:

$$Sn = [1-(y-x)(a/D)^{0.5}(c_p/r)\,dT/dx]^{-1}. \qquad (5)$$

Equation (4) has no empirical constants. It was verified with a limited number of data taken with n-propanol–water mixtures (see line 9 in Table 1).

Based on the effect of composition on the vapour–liquid exchange mechanism for nucleate pool boiling at a heated surface, Thome [14] derived the following analytical expression for the wall superheat:

$$\Delta T_m = \Delta T_i/Sn^{7/5}. \qquad (6)$$

Both equations (4) and (6) include the properties and the liquid mass diffusivity D of a working medium. As indicated in ref. [1], "since there is no simple, general equation for predicting D over a wide range of binary mixture systems, it is difficult to utilize these equations".

The conclusion drawn from the foregoing is that predictive equations (of fairly broad generality) to determine nucleate pool boiling heat transfer coefficients for binary mixtures are lacking in the literature. The aim of this study therefore is to derive such a correlation.

CORRELATION DERIVATION

Since the subject being dealt with is too complex to tackle analytically, dimensional analysis was used to derive a correlation to predict wall superheat during nucleate pool boiling of a binary mixture. As is already known, the dimensionless numbers which fully characterize two-phase flow are numerous and it is practically impossible to take all of them into account when correlating data. For the selection of the dimensionless numbers used in the correlation, therefore, the following empirical procedure was adopted. The work of Stephan and Körner [8] indicates that the ratio of this wall superheat to the ideal wall superheat for a given heat flux and a binary mixture is a function of pressure and the mass transfer driving force $|y-x|$. The superheat ratio in fact, shows the non-ideality of a mixture. Accordingly, in order to describe this non-ideality, it is considered justifiable to use an expression analogous to the excess proper Gibbs function [15], as given below:

$$b_1 = (1-x)\ln\frac{1.01-x}{1.01-y} + x\ln\frac{x}{y}. \qquad (7)$$

Table 1. Ranges of data used

No.	Binary mixture	P/P_c	q (kW m^{-2})	x	n	Ref.
1	acetone–n-butanol	0.021–0.21	100	0.02–0.80	38	9
2	benzene–toluene	0.020–0.20	100	0.1–0.90	20	9
3	ethanol–cyclohexane	0.015–0.15	100	0.001–0.91	66	9
4	n-heptane–methylcyclohexane	0.036–0.36	100	0.10–0.94	35	9
5	methanol–benzene	0.012–0.19	100	0.055–0.91	54	9
6	methylethylketone–toluene	0.024–0.24	100	0.09–0.90	36	9
7	acetone–water	0.021–0.21	100	0.001–0.88	47	9
8	i-propanol–water	0.018–0.18	100	0.002–0.93	31	9
9	n-propanol–water	0.021	100–400	0.029–0.80	21	10
10	ethanol–water	0.016	100–200	0.040–0.95	15	1, 11
11	ammonia–water	0.07	5.8–11.6	0.21–0.85	9	16
12	SF_6–CF_3Br	0.59–0.95	10	0.047–0.91	10	17
13	acetone–methanol	0.021	100	0.039–0.84	6	11

For highly non-ideal mixtures, acetone–water mixtures for example, and at low liquid concentrations of volatile component, y/x can be considerably large. For such a case, equation (3) fails to predict accurately the heat transfer coefficient. It was for this reason then that y/x was also taken into account as a correlating parameter. Thus the dimensionless numbers or expressions under consideration are the previously mentioned superheat ratio, b_1 as given by equation (7), y/x, $|y-x|$ and the reduced pressure of the volatile component. The results of the correlation should yield the heat transfer coefficient for the pure volatile (i.e. lower boiling point) component and for the pure non-volatile (i.e. higher boiling point) component at $x = 1$ and $x = 0$, respectively. For an azeotropic mixture, ΔT_m is assumed to be equal to ΔT_i at its azeotrope, since such a mixture acts in the manner of a pure or single component fluid at this particular point.

The ranges of operating condition for the data considered to establish the correlation are summarized Table 1. Most of these data were taken from Körner's thesis [9]. The data quoted in ref. [17] for SF_6/CF_3Br mixtures were obtained at elevated pressures. The data of Shakir for ethanol–water mixtures (number 10 in Table 1) were quoted from ref. [1]. The relevant phase equilibrium data for this mixture were taken from ref. [11]. For the other mixtures, these data were available from the references cited in Table 1. For the data of ref. [16] for ammonia–water mixtures (number 11 in the table), the pressure for the tests was not reported. In comparing the phase-equilibrium data given for these tests with those given in ref. [18], it was concluded that this pressure was about 0.80 MPa.

The correlation obtained is

$$\frac{\Delta T_m}{\Delta T_i} = [1+(b_2+b_3)(1+b_4)][1+b_5] \tag{8}$$

where

$$b_2 = (1-x)\ln\frac{1.01-x}{1.01-y} + x\ln\frac{x}{y} + |y-x|^{1.5} \tag{9}$$

$$b_3 = 0 \quad \text{for} \quad x \geqslant 0.01 \tag{10}$$

$$b_3 = (y/x)^{0.1} - 1 \quad \text{for} \quad x < 0.01 \tag{11}$$

$$b_4 = 152(P/P_c)^{3.9} \tag{12}$$

$$b_5 = 0.92\,|y-x|^{0.001}(P/P_c)^{0.66} \tag{13}$$

and

$$\frac{x}{y} = 1 \quad \text{for} \quad x = y = 0. \tag{14}$$

ΔT_i in equation (8) and, h, the heat transfer coefficient, are expressed by equations (2) and (3), respectively. ΔT_m in the latter is now given by equation (8).

Firstly, using the data taken with mixtures quoted in lines 1–5, 7, 8 and 12 in Table 1, the correlation was established. Thereafter it was verified with the data taken with mixtures mentioned in lines 6, 9–11 and 13 in the table.

The results of comparing equation (3) with all the data taken into consideration are shown in Fig. 1. The numbers in this figure correspond to those in Table 1. ΔT_i in equation (8) was determined using the measured values of ΔT_1 and ΔT_2. Equation (3) predicts the heat transfer coefficient from the data accurate to 30% for 97% of the time. The rest of the time, the equation is accurate to 38.8%. The r.m.s. error for the 388 data analysed is 14.8%. It follows from the Introduction then, that these accuracies seem to be quite acceptable with the present state of the art.

SUMMARY AND CONCLUSIONS

A correlation was established to predict nucleate pool boiling heat transfer coefficients for binary mixtures. The 388 data taken with 13 binary mixtures (including highly non-ideal mixtures) were considered; the range of operating conditions for the data are: $P/P_c = 0.015$–0.95; $q = 5.8$–$400\ \mathrm{kW\ m^{-2}}$; $x = 0.001$–0.95. The correlation predicts the heat transfer coefficient from these data accurate to 30% for 97% of the time. The r.m.s. error for the 388 data analysed is 14.8%. With the present state of the art, these errors are considered satisfactory.

The correlation presented does not include either an empirical constant different for each binary mixture or the

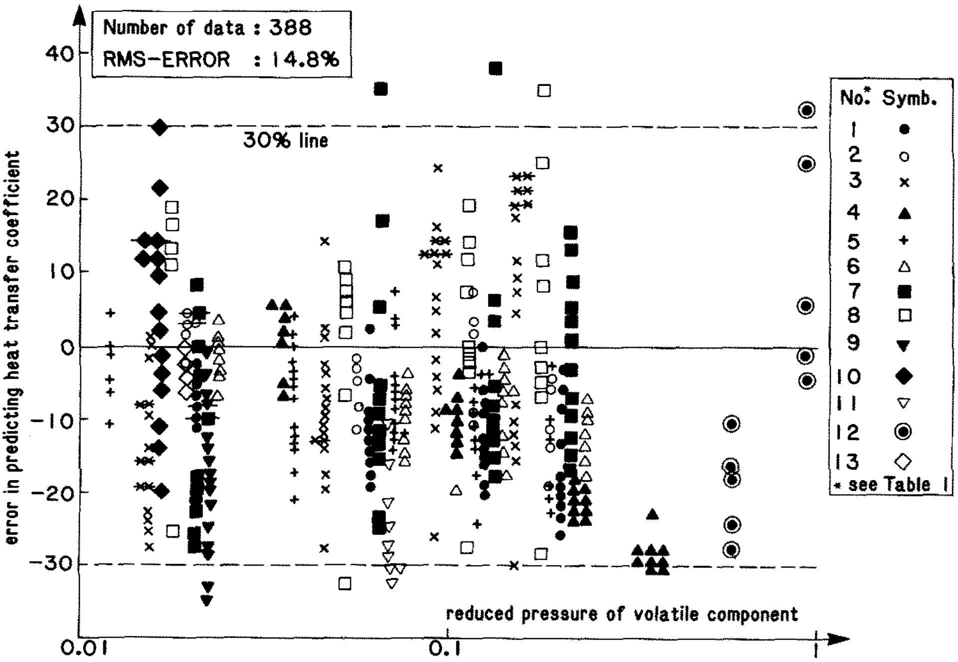

FIG. 1. Errors in predicting heat transfer coefficients.

properties of a binary mixture with the exception of those for phase equilibrium. It was also verified with the data taken at elevated pressures. Accordingly, it appears to be of practical significance for practising engineer.

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A simple method of dimensioning straight fins for nucleate pool boiling

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INTRODUCTION

Many methods have been proposed to enhance nucleate pool boiling; methods related to surface structures seem most promising. For a detailed literature survey on the subject, the reader is directed to [1, 2]. Only the data in the literature relevant to the present study will be briefly mentioned in this paper.

Fundamental work has shown that, for the enhancement of nucleate pool boiling, the surface of a tube should, in general, have either interconnected re-entrant grooves or a porous coating [1]. At present, both the dimensioning of the structure of an enhanced surface and the evaluation of its performance are carried out by experiment, since no predictive correlating equations of broad generality exist in the literature [1, 2].

Four types of commercially available and patented surfaces applied to circular tubes are mentioned. These are the High-Flux, the Thermoexcel-E, the Gewa-T and the ECR40 surfaces.

The High-Flux surface consists of a porous, metallic matrix which is bonded to a metallic substrate. The surface layer is about 0.25–0.50 mm thick with a void fraction of 50–65% and contains a multiplicity of cavities or pores which function as sites for the generation of vapour bubbles [3]. The Thermoexcel-E surface has continuous tunnels and isolated pores on top of the tunnels. The construction is manufactured by bending the ridges of rugged microfins. Typical values for the geometry of this surface are: tunnel width, 0.2 mm; depth, 0.4 mm; pitch, 0.55 mm; pore diameter, 0.1 mm; and number density of pores, 260 cm^{-2} [2]. The Gewa-T surface is obtained by flattening the fin tips of a integral fin tube [1, 2]. It is in fact identical to the ECR40 surface, excepting that the flat crests of the fins in the latter are provided with porous plates [1, 2].

Yilmaz and Westwater [4] have measured the performances of copper circular tubes with these four surfaces and also the performance of a plain copper tube during saturated nucleate pool boiling of *p*-xylene and isopropyl alcohol at atmospheric pressure. In accordance with the data obtained by these investigators, the wall superheat required for boiling on these tubes is much lower than that on a plain tube. For a given heat flux and at low wall superheats, the heat transfer coefficients for these tubes are significantly higher than the plain-tube heat transfer coefficient. At high wall superheats however, the improvements in these heat transfer coefficients appear to be quite moderate. For isopropyl alcohol, the ratios of these heat transfer coefficients to the plain tube heat transfer coefficient vary between about 1.7 and 6.9 for a heat flux of 60 kW m^{-2}, and between about 1.3 and 3.3 for a heat flux of 340 kW m^{-2}. The wall superheat for the plain tube is about 9.8 and